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WAKE STRUCTURE IN HIGH-SPEED FLOW OF A RAREFIED PLASMA OVER A BODY

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In the study of qualitative features of flow of a rarefied plasma over bodies in ionospheric aerodynamics, the problem of flow behind a two-dimensional plate is often considered. The formulation of this problem and its relation to flow over real objects was considered in detail in [1]. This model problem has been analyzed in a number of papers using two main approaches: description of the flow with the help of the similarity solution found in [2, 3], and numerical solution of the equations of plasma motion [4-7]. A review of the main results obtained by the two methods can be found in [1, 6]. This paper gives a numerical solution of the problem of transverse supersonic flow over a flat plate. The plasma is assumed to be collisionless and is described by the kinetic equation with a self-consistent field. The particle-in-cell method is used to solve the kinetic equation. In contrast with most numerical calculations previously performed [4-6], the present paper considers the case, of greater practical interest, of flow over a body whose dimension R is much greater than the Debye radius D_i in the unperturbed plasma. Practically all the known results for this case have been obtained using the similarity solution [2, 3], which is not valid, however, in the entire region of unperturbed flow, and therefore does not give a complete solution to the problem. Individual numerical calculations (see [7]) do not add much to the similarity analysis, since they refer to a very narrow range of the flow parameters. The main emphasis in the present paper is the study of wake structure behind a flat plate and plasma instability in the wake. The computations were performed in a wide range of variation of the ratio $\beta = T_e/T_i$, and one can follow the processes of ion acceleration, interaction of the accelerated group of ions with the plasma, development of beam-type instability [1, 8], and formation and decay of the turbulent wake. The qualitative wake structure features discussed below are also found, of course, in plasma flow over actual three-dimensional bodies.

1. The formulation of the problem adopted here was discussed in detail in [1]. We consider steady-state plasma flow near a two-dimensional plate. The plasma velocity V far from the plate is directed normal to it and satisfies the inequalities

$$\sqrt{T_e/m_i} \ll V \ll \sqrt{T_e/m_e}. \tag{1.1}$$

The first inequality indicates that the flow is supersonic. Behind the plate a cylindrical region remains free from ions, with a cross section equal to the plate area. In this region, as in vacuum, the plasma expands. In a coordinate system in which the plasma is at rest far from the body, the filling up of the cylindrical cavity is an unsteady process which accurately corresponds to free expansion of a plasma into vacuum [2, 3, 9] until collision occurs between the two particle fluxes reaching the cavity from opposite directions. In the coordinate system fixed in the body, the flow is steady-state and is a superposition of a transverse expansion and a longitudinal drift with velocity V. Thus, the wake structure in successive cross sections at different distances z from the plate corresponds to successive stages of unsteady filling of the cavity by the plasma. One flow is obtained from the other by the substitution z→Vt. This analogy, which is well known from hypersonic aerodynamics [10], and is discussed in detail in [1, 5], is used extensively in the calculations below.

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The second of inequalities (1.1) indicates that the speed of motion of the plasma relative to the plate is much less than the thermal speed of the electrons. With this condition one can simplify the problem and consider the electron distribution as being in local equilibrium in a self-consistent electric field; i.e., one can put

$$n_e = n_0 \exp(e\varphi/T_e), \quad (1.2)$$

where n_e is the electron density and φ is the self-consistent field potential.* The latter is determined by solving Poisson's equation

$$\Delta\varphi + 4\pi e(n_i - n_e) = 0, \quad (1.3)$$

where $n_i = \int f_i(\mathbf{r}, \mathbf{v})d\mathbf{v}$ is the ion density; $f_i(\mathbf{r}, \mathbf{v})$, ion distribution function; and n_e, given by Eq. (1.2).

The ion distribution function can be found by solving the collisionless kinetic equation

$$V\frac{\partial f}{\partial z} + v_x\frac{\partial f}{\partial x} - \frac{e}{m_i}\frac{\partial \varphi}{\partial x}\frac{\partial f}{\partial v_x} = 0. \quad (1.4)$$

In writing Eq. (1.4) it is assumed that the velocity **V** is directed along the z axis and the plate is of infinite extent in the y direction. The latter assumption was adopted for simplicity (the problem with cylindrical symmetry would be closer to reality) and allows one to neglect the derivatives of the distribution function and the potential with respect to y. We note that for large flow velocities V the electric field component $E_z = -\partial\varphi/\partial z$ is also small compared with $E_x = -\partial\varphi/\partial x$, and is therefore omitted in the equations as written. The function $f(x, z, v_x)$, determined by Eq. (1.4), is obtained from the original ion distribution function $f_i(\mathbf{r}, \mathbf{v})$ by integration over the velocity components v_y and v_z, which do not appear in the equation.

To solve system (1.2)-(1.4) the particle-in-cell method [11] is used. Since the electrons are assumed to have a Boltzmann distribution, the modeling is carried out for only one kind of particle, the ion. However, then the equation for the potential φ becomes nonlinear.

Integration of the system was carried out for the following boundary conditions. For $z > 0$ in the plane of symmetry $x = 0$ the mirror reflection condition was applied for particles, and the electric field was assumed to be zero. The same conditions were applied at the boundary of the computational region $x = R_0$. At $z = 0$ in the region $0 \le x \le R$ there are no particles, the region $R < x \le R_0$ is filled uniformly with plasma, and the ions there have a Maxwellian distribution with respect to v_x. A numerical solution of this problem was obtained for the following region of values of variables and parameters:

$$0 \leqslant z \leqslant 250V\omega_{pi}^{-1}; \quad 0 \leqslant x \leqslant R_0; \quad R = 100D_i;$$
$$200 \leqslant R_0/D_i \leqslant 600; \ 1 \leqslant T_e/T_i \leqslant 200.$$

2. The results of the computations can be conveniently represented by the use of dimensionless variables. The x coordinate is referenced to the ion Debye radius of the unperturbed plasma D_i, and the z coordinate is referenced to the length $V\omega_{pi}^{-1}$, equal to the Debye radius D_i, multiplied by the Mach number of the unperturbed flow. The unit of velocity was chosen to be $\sqrt{T_i/m_i}$, the unit of potential was T_i/e, the unit of electric field intensity was $\sqrt{4\pi n_0 T_i}$, and the unit of density was n_0. Energy is expressed in terms of the energy of ions of the unperturbed plasma column of unit cross section with a length equal to the Debye radius.

We consider first the case of a single-temperature plasma ($T_e = T_i$). For small z the plasma motion in the x direction is an ordinary expansion into vacuum. For $z \sim 20$ interaction of the streams reaching the wake from opposite sides becomes noticeable, and the expansion is slowed. The ion velocity V_{0x}, averaged over the whole range of values of x, the displacement of the center of mass ΔX_0, and the quantity $R - X_m$ (X_m is the coordinate of the large ion front) are shown in Fig. 1 as a function of the coordinate z. In the initial stage the mean-mass motion occurs with constant acceleration [9], and then for $z \sim 40$ motion at constant speed is established. The broken line in Fig. 1 shows the same relationships for a free expansion into vacuum.

The expansion into vacuum is accompanied by acceleration of the ions in the electric field. The total ion component energy ε_i here increases, while the electronic component ε_e decreases linearly [9] with time. The same thing occurs in the wake at the initial stage of filling, when the plasma flux does not yet "know" that there is another flux from the opposite side of the wake (Fig. 2). The ion acceleration process continues, even after collision of the flows, although it becomes significantly slower than in the free expansion (the latter corresponds to the broken lines in Fig. 2).

*We note that at sufficiently large distances from the body the filling of the rarefied zone occurs with a speed on the order of the thermal speed of the ions. In this region Eq. (1.2) holds for any body speed.

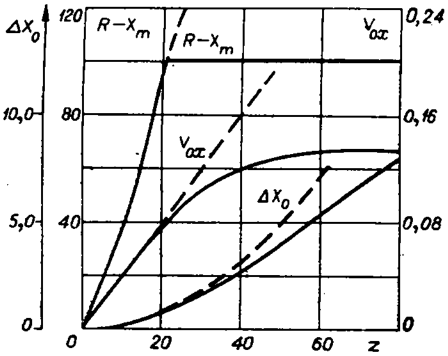


Fig. 1

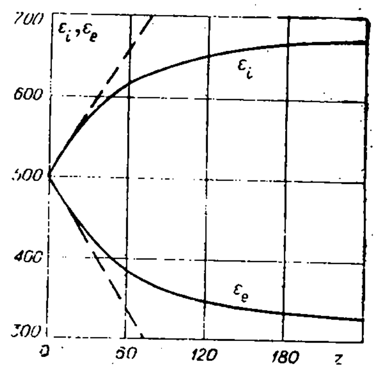


Fig. 2

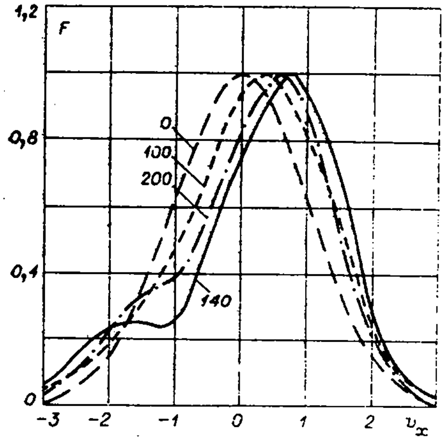


Fig. 3

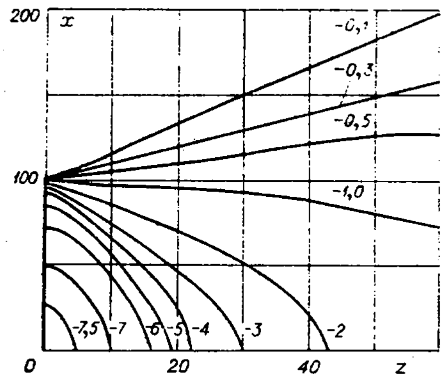


Fig. 4

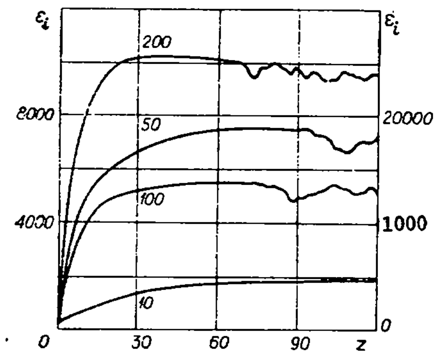


Fig. 5

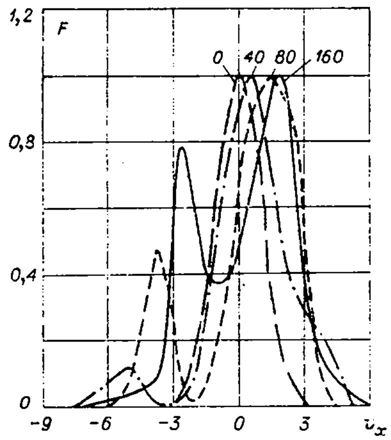


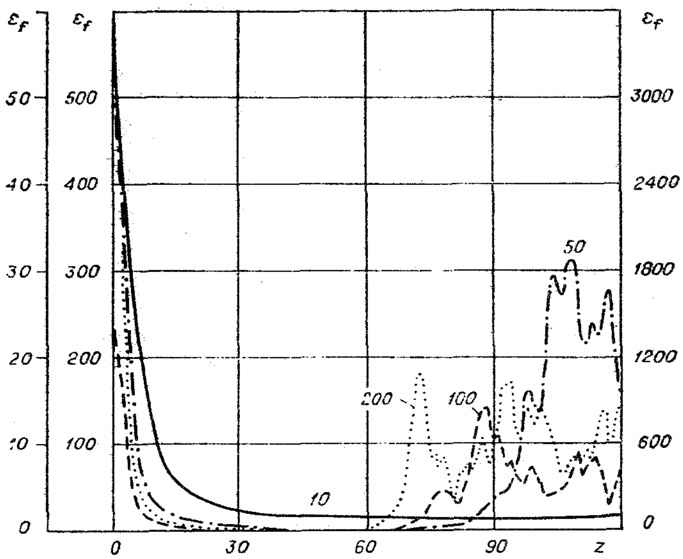
Fig. 6

The interaction of the two streams appreciably alters the form of the ion distribution function. Figure 3 shows the distribution function, integrated with respect to x,

$$F(z, v_x) = \int_0^{R_0} f(x, z, v_x)\, dx,$$

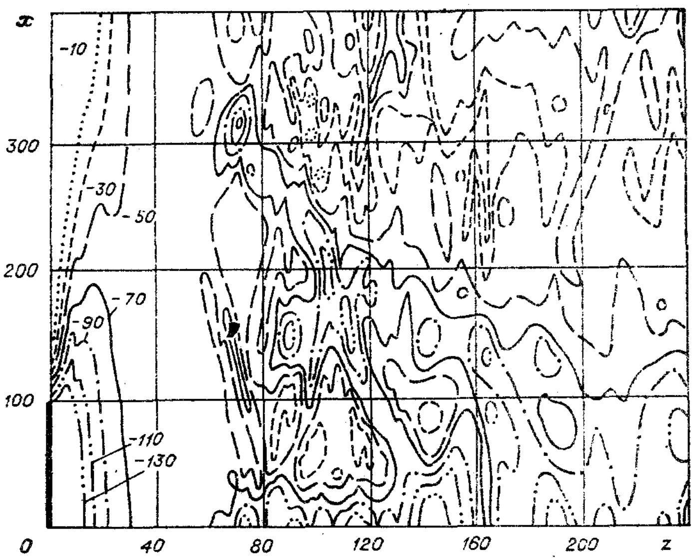
at different cross sections of the wake (the cross-sectional coordinate is shown on each curve; the function is normalized to the value at maximum; the solid curve corresponds to $R_0 = 400$, and the other curves to $R_0 = 600$). With increase of z there is a significant redistribution of the initial Maxwellian distribution function. Initially there is an increase in the number of particles with large negative velocities (i.e., directed away from the center of the wake), and then a second maximum appears in the ion distribution.

Figure 4 shows a family of equal potential lines in the xz plane. The parameter is the potential. For $T_e = T_i$ the potential is a monotonic function of the two variables. The energy density of the self-consistent field



ε_f
ε_f
ε_f
50
40
30
20
10
0
500
400
300
200
100
3000
2400
1800
1200
600
0
50
200
100
10
0
30
60
90
z

Fig. 7



x
300
200
100
0
-10
-30
-50
-70
-90
-110
-130
40
80
120
160
200
z

Fig. 8

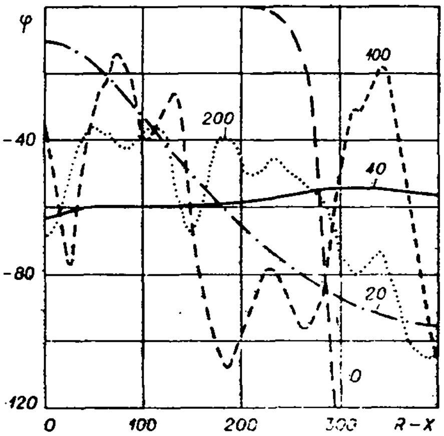


Fig. 7

increases rapidly with increase of z, and on the average is only an insignificant part of the particle energy density. We note, incidentally, that the absence of potential oscillations for $T_e = T_i$ is also confirmed from the results of [7], and is associated with the great attenuation of the ion sound in this case.

We now consider the case of an unequal-temperature plasma, with $T_e \neq T_i$. The qualitative picture of the flow in this case is retained, in the main. However, a number of quantitative flow characteristics show a strong dependence on the nonisothermal parameter $\beta = T_e/T_i$.

With increase of β there is an increased speed of wake filling by the plasma and in the rate of collisionless exchange of energy between the electron and ion components. The total energy transferred from the electrons to the ions increases linearly with β, over a wide range of values of β, and is approximately one-third of the initial electron energy. The ion component energy is shown in Fig. 5 as a function of z, for different values of β. The left scale is for $\beta = 10$ and 50, and the right scale is for $\beta = 100$ and 200.

For $T_e \gg T_i$ there is a very abrupt redistribution in the ion distribution function during the wake filling process. Figure 6 shows the distribution function $F(z, v_x)$, integrated over x, for $\beta = 10$, at various wake sections. It can be seen that even for $z = 40$ the ion distribution has a pronounced bimodal form.

It was shown in [1] that one should expect beam-type instability in the filling of spaces behind a flat plate. Naturally, the development of instability is facilitated with increase in the temperature ratio β. Figure 7 shows the increase in electric field energy with z, resulting from excitation of ion-acoustic waves, for various values of β. The extreme left scale corresponds to $\beta = 10$, the left scale to $\beta = 50$, and the right scale to $\beta = 100$ and 200. The instability increment, evaluated from the calculations, is on the order of $0.01\omega_{pi}$ for $\beta = 10$ and $0.02\omega_{pi}$ for $\beta = 200$. Figure 8 shows equal potential lines in the xz plane for $\beta = 200$. The picture differs qualitatively from that shown in Fig. 4 for the case $\beta = 1$. For $z > 50$ the potential distribution has the character of disordered fluctuations whose amplitude first increases with increase of z, and then slowly decays. This can be seen very well in Fig. 9, which shows the potential distribution at various cross sections of the wake for $\beta = 200$.

For smaller β the potential oscillations have smaller amplitude and are more ordered in nature, so that one can speak of a qualitative agreement of the picture obtained with the results of [4], which considered plasma flow with cold ions near a sphere of radius $R \sim D_e$.

Comparison of the computed results with the similarity solution shows that the latter gives a practically exact quantitative description of the flow at distances from the body where the interaction of the fluxes arriving from opposite sides of the wake does not play an appreciable part. This region decreases with increase of β. For large z the flow character is determined by instability, and the picture of instability development is in qualitative agreement with the analysis made in [1].

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A CALCULATION OF THE PARAMETERS OF THE HIGH-SPEED JET FORMED IN THE COLLAPSE OF A BUBBLE

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As is known, the collapse of vapor bubbles in a liquid can cause the intensive destruction of solid boundary surfaces. Experimental and theoretical investigations of bubble collapse have led to the conclusion that the surface of a bubble can deform and a liquid jet directed toward the solid surface can form in the process [1, 2]. In the theoretical reports [3, 4] too low jet velocities were obtained, inadequate to explain the destruction of the surface in a single impact. In [5] it was found as a result of numerical calculations that the formation of jets possessing enormous velocities is possible. It was also found that two fundamentally different schemes of jet formation are possible in the collapse of a bubble near a wall. The transition from one scheme to the other occurs upon a relatively small change in the initial shape of the bubble. In the present report we investigate the case of sufficiently small initial deformations of a bubble when the region occupied by the bubble remains simply connected during the formation of the jet; i.e., the separation of a small bubble from the bubble does not occur. In the case of the second scheme of bubble collapse near a wall the connectedness of the free boundary is disrupted and a small bubble separates off during the formation of the jet.

In an ideal incompressible liquid, bounded by a plane solid surface and stationary at infinity, there is a bubble. At the boundary of the bubble the liquid pressure is $p=0$ and at infinity $p=p_\infty$. At the starting time $t=0$ the shape and position of the bubble are given. It is required to determine the motion of the liquid and the shape of the bubble boundary S at $t>0$.

The motion of the liquid was calculated numerically on a BÉSM-6 computer using the method of calculating the potential motions of a liquid with free boundaries suggested in [6]. In the axisymmetric problem the bubble contour is represented with the help of interpolation on a large number of reference points (from

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